Summary

HL - Calculus

Subject	Year	Start date	Duration
Mathematics: applications and interpretation	IB2	Week 2, February	6 weeks

Course Part

Description

In this unit you will learn how to differentiate and integrate functions as well as solving differential equations.

Inquiry & Purpose

Inquiry / Higher Order Questions

Type	Inquiry Questions
Concept-based	In what applications could volumes of revolutions be used?
Content-based	What are the flaws of numerical methods such as the trapezium rule and Eulers method?

Curriculum

Aims

Develop an understanding of the concepts, principles and nature of mathematics

Appreciate the universality of mathematics and its multicultural, international and historical perspectives

Objectives

Knowledge and understanding: Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.

Syllabus Content

Topic 5: Calculus

SL Content

SL 5.1

Introduction to the concept of a limit.

Derivative interpreted as gradient function and as rate of change.

SL 5.2

Increasing and decreasing functions.

Graphical interpretation of $f'(x) > 0, f'(x) = 0, f'(x) < 0$

SL 5.3

Derivative of $f(x) = ax^n$ is $f'(x) = anx^{n-1}, n \in \mathbb{Z}$

The derivative of functions of the form $f(x) = ax^n + bx^{n-1} + -$ where all exponents are integers.

SL 5.4

Tangents and normals at a given point, and their equations.

SL 5.5

Introduction to integration as anti-differentiation of functions of the form$f(x) = ax^n + bx^{n-1} + \ldots.$, where $n \in \mathbb{Z}, \quad n \neq -1$

Anti-differentiation with a boundary condition to determine the constant term.

Definite integrals using technology.

Area of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x) > 0$.

SL 5.6

Values of x where the gradient of a curve is zero. Solution of x.

Local maximum and minimum points.

SL 5.7

Optimisation problems in context.

SL 5.8

Approximating areas using the trapezoidal rule.

AHL Content

AHL 5.9

The derivatives of $\sin x, \cos x, \tan x, \mathrm{e}^x, \ln x, x_n$ where $n \in \mathbb{Q}$

The chain rule, product rule and quotient rules.

Related rates of change.

AHL 5.10

The second derivative.

Use of second derivative test to distinguish between a maximum and a minimum point.

AHL 5.11

Definite and indefinite integration of x^n where $n \in \mathbb{Q},$ including $n = -1, \sin x, \cos x, \dfrac{1}{\cos^2 x}$ and e^x

Integration by inspection, or substitution of the form $\int f(g(x))g'(x)\mathrm{d}x$

AHL 5.12

Area of the region enclosed by a curve and the x or y-axes in a given interval.

Volumes of revolution about the x- axis or y- axis.

AHL 5.13

Kinematic problems involving displacement s, velocity v and acceleration a.

AHL 5.14

Setting up a model/differential equation from a context.

Solving by separation of variables.

AHL 5.15

Slope fields and their diagrams.

AHL 5.16

Euler's method for finding the approximate solution to first order differential equations.

Numerical solution of $\dfrac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$

Numerical solution of the coupled system $\dfrac{\mathrm{d}x}{\mathrm{d}t} = f_1(x, y, t)$ and $\dfrac{\mathrm{d}y}{\mathrm{d}t} = f_2(x, y, t)$

AHL 5.17

Phase portrait for the solutions of coupled differential equations of the form:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = ax + by$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = cx + dy$$

Qualitative analysis of future paths for distinct, real, complex and imaginary eigenvalues.

Sketching trajectories and using phase portraits to identify key features such as equilibrium points, stable populations and saddle points.

AHL 5.18

Solutions of $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = f\left(x, \dfrac{\mathrm{d}x}{\mathrm{d}t}, t\right)$ by Euler's method.

ATL Skills

Approaches to Learning

Thinking

Developing IB Learners

Learner Profile

Inquirers

Knowledgeable

Thinkers